Grade 7
Geometry Grand Finale

Date	Topic	Homework	IXL	Grade
5-30	Regular Polygons	Worksheet 1; Read pp. 2-4	W.10	
5-31	Interior and Exterior Angles	Worksheet 2; Read pp. 8-9	W.9	
6-3	Parallel Lines	Worksheet 3	W.12	
6-4	More Work with Transversals	Worksheet 4	W.13	
6-5	Algebra in the Geometry	Worksheet 5	W.14	
6-6	Slicing Three-Dimensional Shapes	Review Packet		
6-7	Review- IXL due tonight	Study for Test		
6-10	Unit Test	IXL due		

Unit Test Monday, June 10th

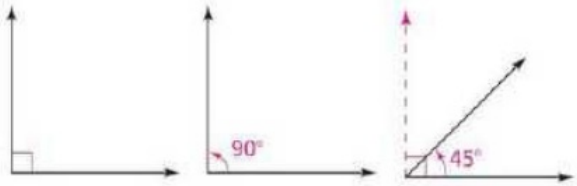
Day 1 – Angles

Read.

Measuring flips and spins involves thinking about an angle as a change in direction called a *rotation*. In mathematics, you measure an angle or a rotation with a unit called the **degree.** Rotation angles are measured from 0 degrees to 360 degrees or more to indicate turns from a small amount to one full turn (and more).

You measure rotation angles in a counterclockwise direction. A rotation angle has an *initial* and a *terminal side*. The initial side is the ray showing the starting direction while the terminal side is the ray showing the ending direction after the rotation.

A one-quarter rotation is 90°. A **right angle** measures 90°. Right angles are commonly marked with a small square. Suppose you draw a ray to divide a right angle into two angles of equal measure. Each angle would be a 45° acute angle.

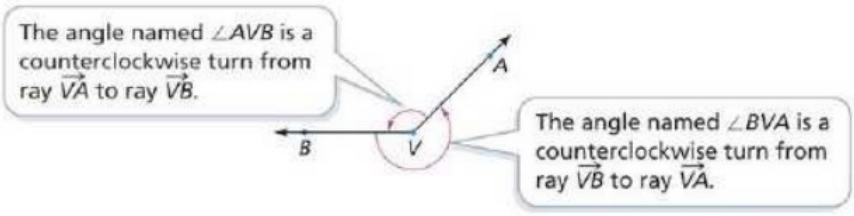


Suppose you draw 89 rays to divide a right angle into 90 angles of equal measure. Each angle would have a measure of 1°.

A rotation of one-half turn defines a *straight angle*. It measures 180°.

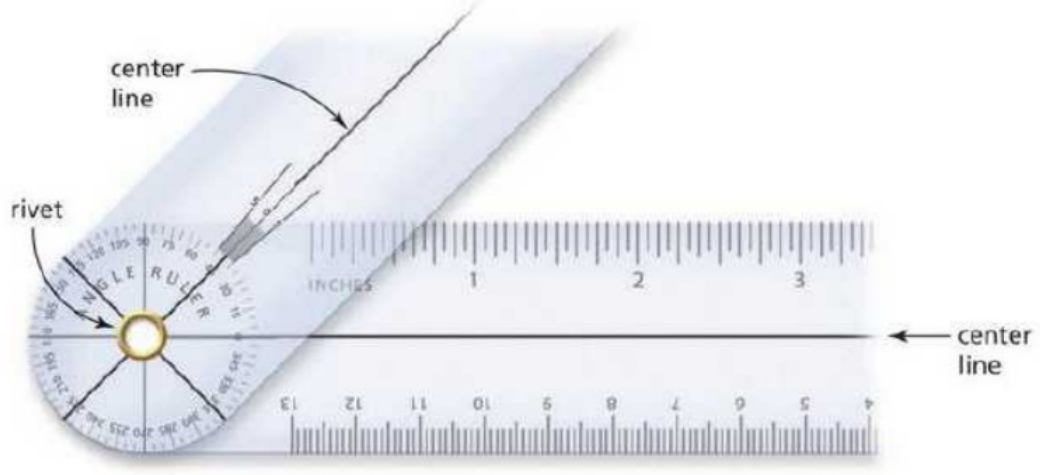
Recall that angles whose measures are less than 90° are called *acute angles*. Angles whose measures are between 90° and 180° are called *obtuse angles*.

The next sketch shows two rays with a common endpoint. The rays are named $\overrightarrow{VA}$ and $\overrightarrow{VB}$. They define two rotation angles.

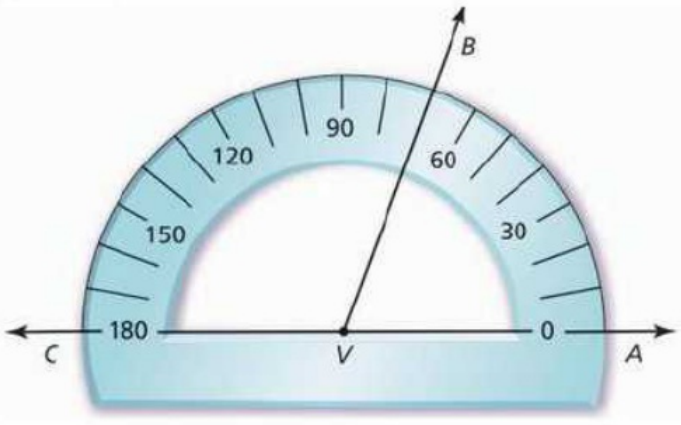


Measuring Angles

One common tool to use for measuring angles is the *angle ruler*. An angle ruler has two arms linked by a rivet. The rivet allows the arms to spread apart to form angles of various sizes. One arm is marked with a circular ruler showing degree measures from 0° to 360°.



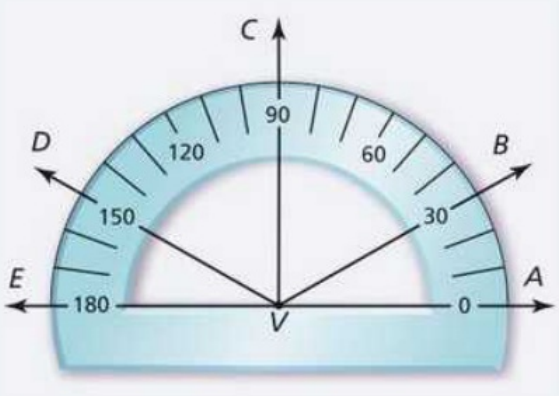
Another tool for measuring angles in degrees is the *protractor*. It is usually semi-circular and has a scale in degrees. The protractor below shows how to measure $\angle AVB$.



Notice in the diagram above that $\angle CVB$ and $\angle AVB$ share a side. Both angles have $\overrightarrow{VB}$ as a side of the angle. Angles that share a side are called *adjacent angles*.

A)

Find measures of the angles shown in the diagram.



1. $\angle AVB$
2. $\angle AVC$
3. $\angle AVD$
4. $\angle BVC$
5. $\angle BVD$
6. $\angle CVD$

B)

If the measures of two angles add to 90°, they are called **complementary angles.** If the measures of two angles add to 180°, they are called **supplementary angles.**

1. Name the pairs of complementary angles in the diagram of Question C.
2. Name the pairs of supplementary angles in the diagram of Question C.

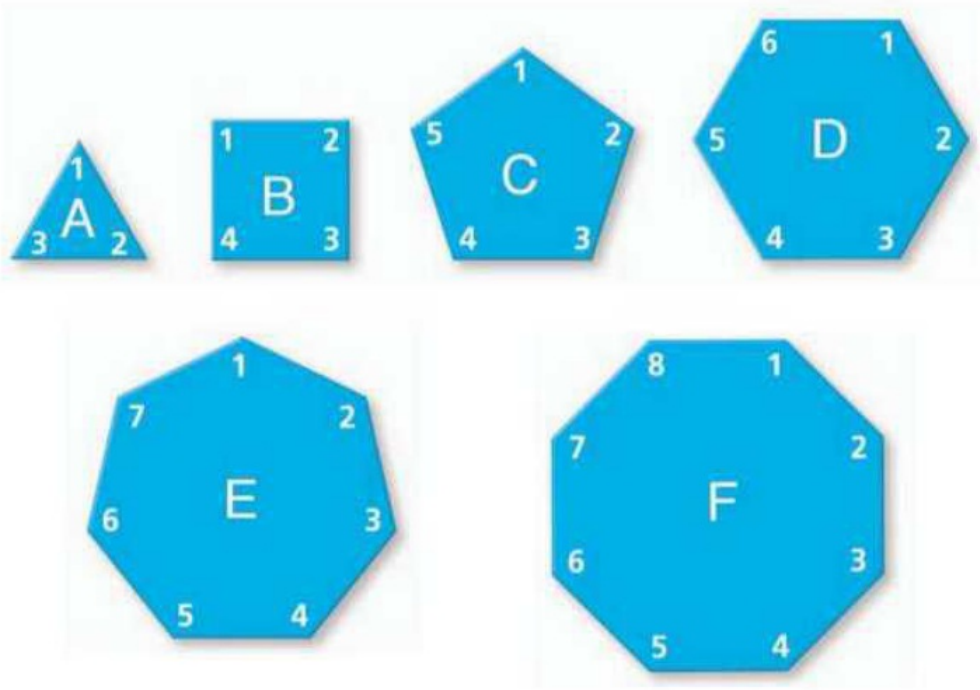
Regular Polygons

You have seen that polygons with the same number of sides can have different shapes. However, there is an important relationship between the number of sides and the angle sum of any polygon. You will develop a formula that relates the number of sides to angle measures.

A **regular polygon** is a polygon in which all of the sides are the same length and all of the angles have the same measure. In an **irregular polygon,** not all of the sides are the same length or not all of the angles have the same measure.

Polygons are named based on the number of sides and angles they have. For example, a polygon with six sides and six angles is called a *hexagon*.

Below are six examples of polygons from the Shapes Set. Study these examples to find a relationship between the number of sides and angles.



A) Measure the angles in each of the polygons using a protractor. Be sure to complete the table below.

Polygon	Number of Sides	Measure of an Angle	Angle Sum
Triangle			
Square			
Pentagon			
Hexagon			
Heptagon			
Octagon			
Nonagon			
Decagon			

B) Find patterns in the table that will help with regular polygons that have even more sides.

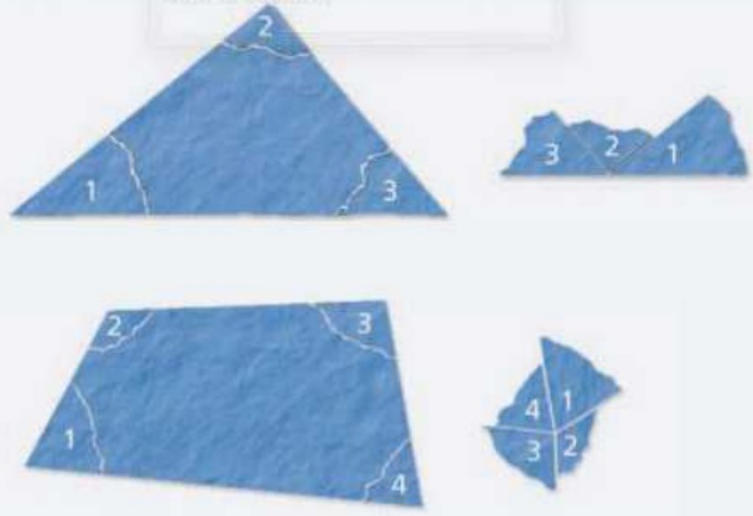
C) Find a pattern relating angle sums to the number of sides in regular polygons.

D) Find a pattern relating measures of individual angles and number of sides in regular polygons.

E)

Devon, Trevor, and Casey tried three different ways to find a formula relating the angle sum of any polygon to the number of sides.

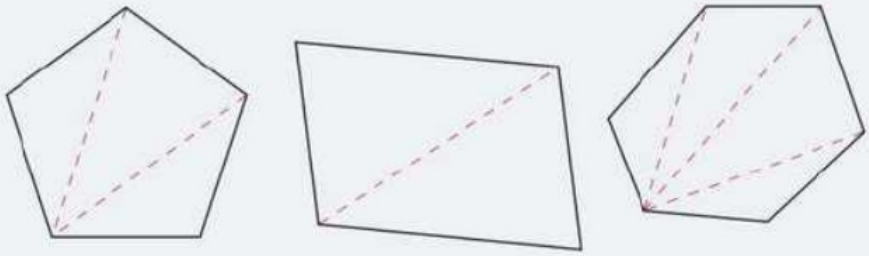
Devon began by drawing irregular triangles and quadrilaterals. Then he tore the corners off of those polygons and 'added' the angles by arranging them like this:



What angle sum does Devon's work suggest for the triangle?
For the quadrilateral?

F)

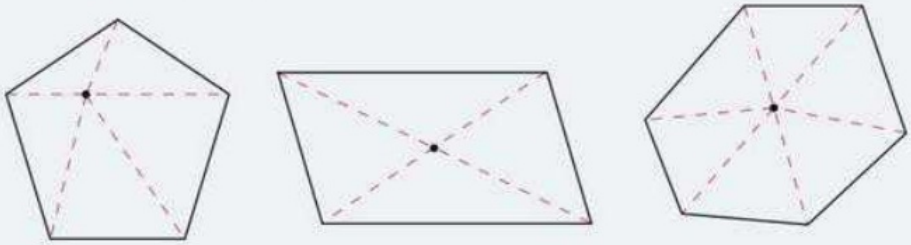
Trevor examined Devon's results from his study of irregular triangles.
This gave him a new idea to study polygons with more sides. He divided some polygons into smaller triangles by drawing diagonals from one vertex.



Describe the relationship between the number of sides of a polygon and the number of triangles formed.

G)

Casey used Devon's discovery about triangles in a different way.
She divided polygons into triangles by drawing line segments from a point within the polygon.

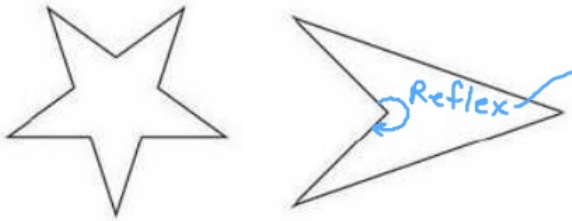


Study Casey's drawings to find the angle sum of each polygon.

Day 2 – Interior and Exterior Angles

Read:

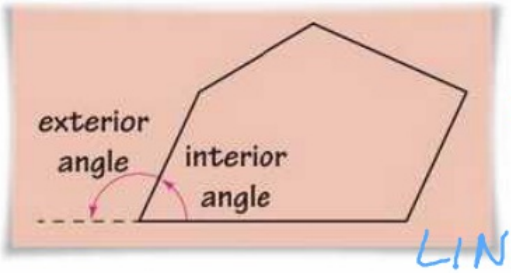
Familiar figures like triangles, parallelograms, and trapezoids are called **convex polygons.** Figures like the star and the arrowhead pictured here are called **concave polygons.**



angle that measures > 180°

For convex polygons it is clear which points are on the inside and which are on the outside. It is also clear how to measure the **interior angles.**

By extending a side of a convex polygon, you can make an **exterior angle** that lies outside the polygon.



"next to" adjacent & add to 180° supplementary

LINEAR PAIR

The figures below show two ways to form exterior angles. You can extend the sides as you move in either direction around the polygon.

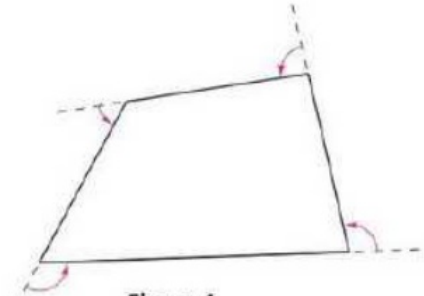


Figure 1
Exterior angles as you move counterclockwise.

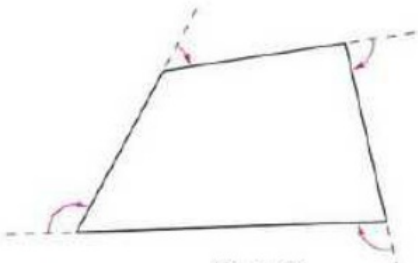


Figure 2
Exterior angles as you move clockwise.

Measuring exterior angles provides useful information about the interior angles of a polygon.

A)

Members of the Columbia Triathlon Club train by bicycling around the polygonal path shown.

They start at vertex *A* and go on to vertices *B*, *C*, *D*, and *E*. Then they return to *A* and start another lap. At each vertex the cyclists have to make a left turn through an *exterior angle* of the polygon.

What is the sum of the left-turn exterior angles that the cyclists make on one full lap around this path?

360°

B) How many pairs of straight angles are there?

5

C) What is the total sum of the straight angles?

$5(180) = 900°$

D) Can you use your answers to parts A-C to find the sum of the interior angles?

$900 - 360 = 540°$

E) Write a formula.

$5(180) - 360$

$n(180) - 360$

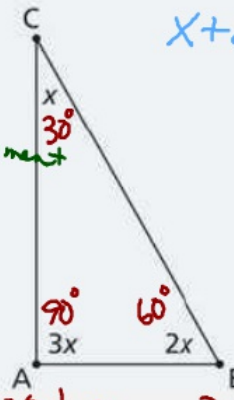
$180n - 360 =$ Sum of the Interior Angles

$n =$ number of Sides

F)

For each of the following triangles write and solve an equation to find the value of x. Use the results to find the size of each angle. Find the supplement of each interior angle.

1.



$x+2x+3x=180$

$\frac{6x}{6}=\frac{180}{6}$

$x=30$

$180-30=150°$ supplement

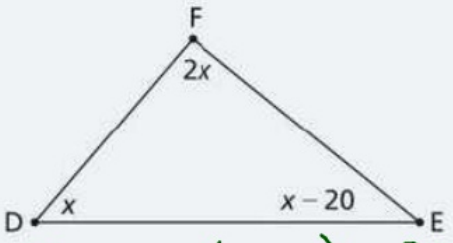
$3x=3(30)=90$

$180-90=90°$

$2x=2(30)=60$

$180-60=120°$

2.



$2x+x+(x-20)=180$

$2x+x+x-20=180$

$4x-20=180$

G) Experiment with strips of Legos of different lengths. Look for combinations of three numbers that will form triangles and combinations that will ***not*** form triangles. Organize your results below.

Octagon → Sum of the Interior Angles.

8 sides

$180n-360$

$180(8)-360=1080$

Do you see any patterns in your results?

6|3

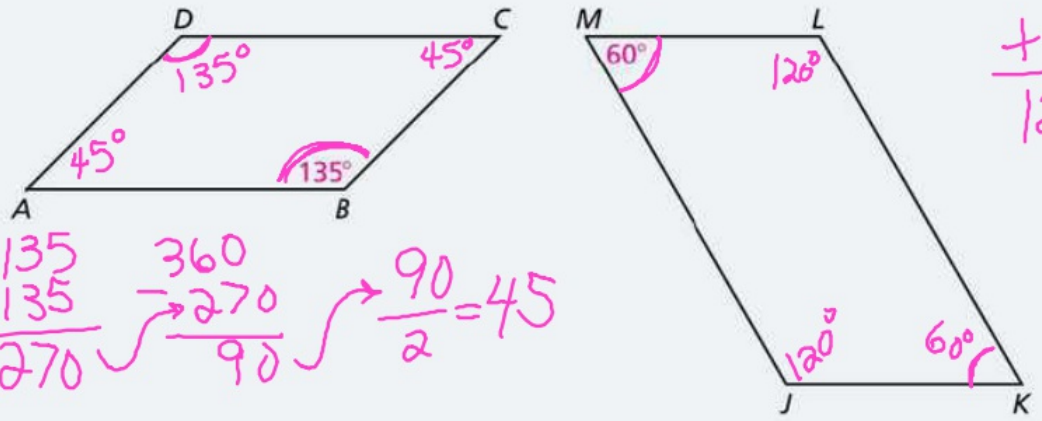
Day 3 – Parallel Lines

Parallel lines are lines in a plane that never meet. They are like railroad tracks, rows of a crop in a field, or lines on notebook paper. They remain the same distance apart and never meet, even if extended forever in both directions.

A **parallelogram** is a quadrilateral in which the pairs of opposite sides are parallel. The shape of a parallelogram is largely set by the angles at which those pairs of sides meet. A parallelogram with four right angles is a **rectangle.**

A)

1. What pattern seems to relate the measures of opposite angles in any parallelogram? they are the same (congruent)
2. What pattern seems to relate the measures of consecutive angles in any parallelogram? They are supplementary consecutive add to 180° in order → in a row
3. Suppose your conjectures in parts (1) and (2) are true. What are the measures of the angles in parallelograms *ABCD* and *JKLM* below?



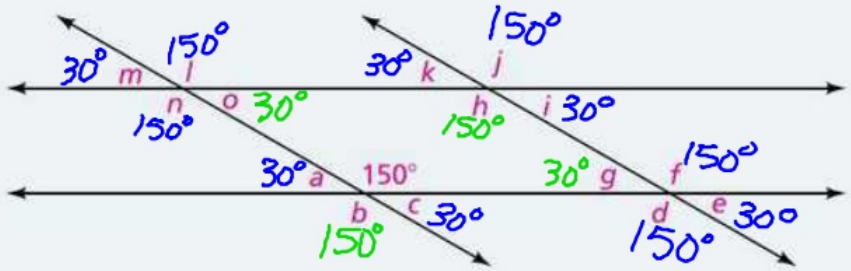
135 + 135 = 270 → 360 − 270 = 90 → 90/2 = 45

60 + 60 = 120 → 360 − 120 = 240

240/2 = 120

B)

Suppose your conjectures from Question A are true. The lines below form a parallelogram.

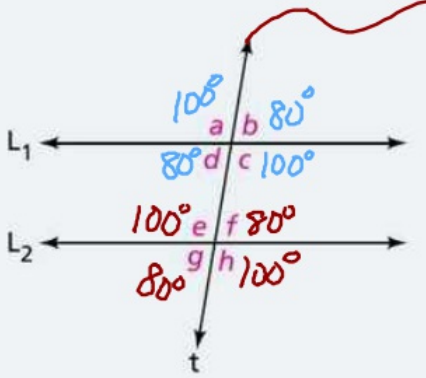


Find the measures of all labeled angles in this diagram. Be prepared to justify each answer.

C)

A line that intersects two other lines is called a **transversal.**

1. From your work in Questions A and B, what can you say about the measures of the eight angles formed by a transversal and two parallel lines?

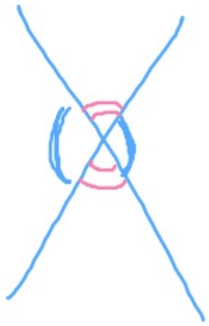


2. Suppose the measure of angle *f* is 80°. What are the measures of the other labeled angles?

vertical angles are always ≅ congruent

D)

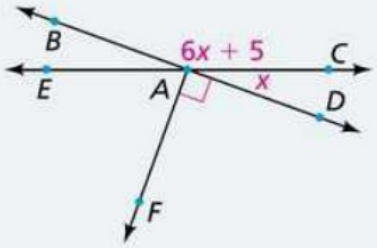
You probably noticed that when two lines intersect four angles are formed. The opposite pairs of angles are called **vertical angles.** For example, in Question C, angles f and g are vertical angles.



1. Name the other pairs of vertical angles in the figure of Question C.
2. Name the pairs of supplementary angles in that figure.
3. What is true about the measures of any vertical angle pair? Explain how you know.

E)

Use what you know about complementary, supplementary, and vertical angles. Write an equation and then find the value of x and the size of each angle in this figure.



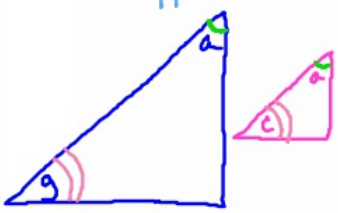
6/4

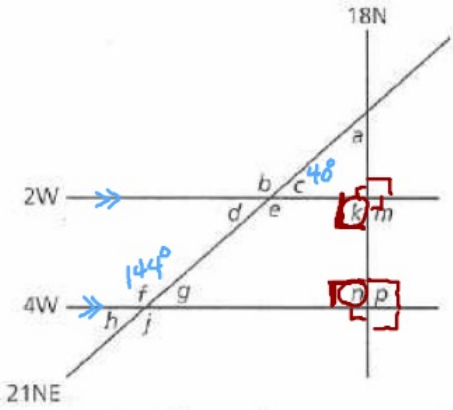
Day 4 – More Work with Transversals

∠g and ∠c are corresponding (always ≅)

1. The local airport is putting in a new terminal and new runways. Part of a diagram of the new runways is shown below. Runways 2W and 4W are parallel.

2W ∥ 4W





PERPENDICULAR

18N ⊥ 2W

18N ⊥ 4W

a. One surveyor measures ∠c to be 40°. Another measures ∠f to be 144°. Explain how you know, without measuring any of the angles, that at least one of the surveyors is incorrect.

∠c + ∠f must = 180°

but 144 + 40 = 184 ≠ 180.

b. What has to be true about the relationship between runway 18N and runways 2W and 4W in order for ∠k and ∠n to have the same measure? Explain using the relationships of angles around transversals.

≅ and +180

90° They must be perpendicular.

c. Describe the relationship between the triangle that includes ∠a and ∠c, and the triangle that includes ∠a and ∠g. Explain how you know.

The triangles are similar

because they have the same angle measure

d. If $m\angle c = 40°$, then what is $m\angle g$? Explain how you know.

40° CORRESPONDING

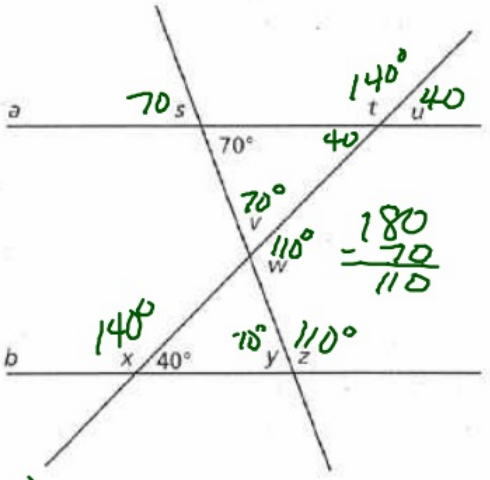
70 + 40 = 110

180 − 110 = 70

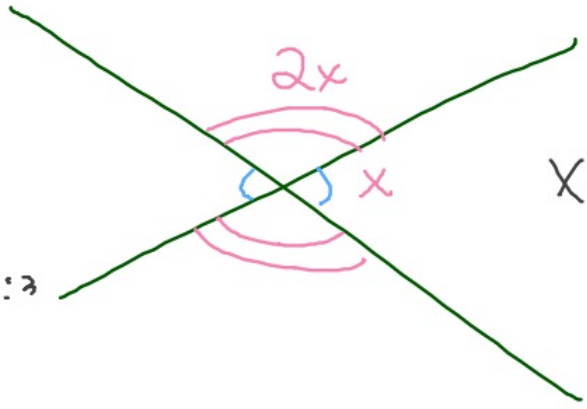
180 − 40 = 140

Find the measure of the angle. Lines a and b are parallel.

1. $\angle s$ 70° (vertical)
2. $\angle t$ 140° (linear pair)
3. $\angle u$ 40° (corresponding)
4. $\angle v$ 70° (triangle)
5. $\angle w$ 110° (linear pair)
6. $\angle x$ 140° (corresponding with t)
7. $\angle y$ 70° (Corresponding with s)
8. $\angle z$ 110° (Linear pair with y)



180 − 70 = 110



$180 = 3x$

$\Rightarrow x = 180 : 3$

$\Rightarrow x = 60$

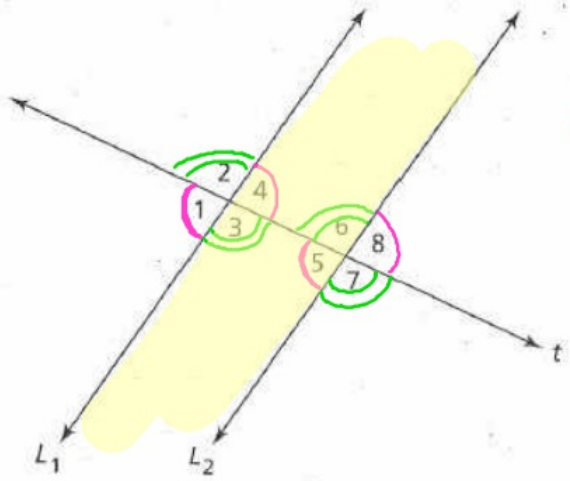
$x + 2x = 180$

$\frac{3x}{3} = \frac{180}{3}$

$x = 60$

Day 5 – Algebra in Geometry

Lines L_1 and L_2 are parallel.



∠1 and ∠8 are alternate exterior angles

∠3 and ∠6 are alternate interior angles.

1. Name two pairs of alternate interior angles.
∠3 and ∠6 ∠4 and ∠5

2. Name four pairs of vertical angles.
∠1 and ∠4 ∠5 and ∠8 ∠6 and ∠7 ∠2 and ∠3

3. Name four pairs of corresponding angles.
∠1 and ∠5 ∠4 and ∠8 ∠2 and ∠6 ∠3 and ∠7

4. Name four angles that are supplementary to ∠4.
∠2, ∠3, ∠6, ∠7

5. The measure of ∠1 is 80°. Find the measures of the other angles.
$m\angle 1 = m\angle 4 = m\angle 5 = m\angle 8 = 80°$
$m\angle 2 = m\angle 3 = m\angle 6 = m\angle 7 = 100°$

$$\begin{array}{r} 180 \\ -\ 80 \\ \hline 100 \end{array}$$

6. **Multiple Choice** What is the reason that ∠5 and ∠6 do not form a pair of alternate interior angles?

A. They are not alternate angles.

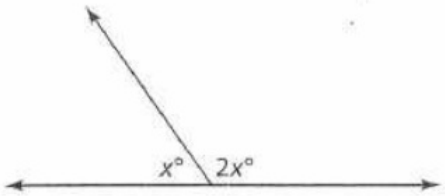
B. They are not interior angles.

C. They share the same vertex.

D. They are not supplementary.

7.

Find the value of x.



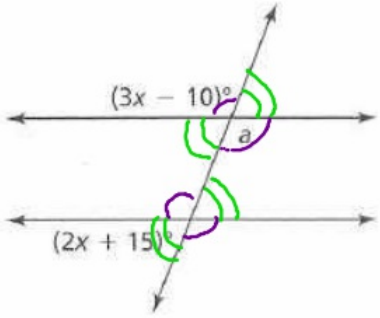
$x + 2x = 180$

$\frac{3x}{3} = \frac{180}{3}$

$x = 60$

8.

Two parallel lines are cut by a transversal.



Find the measure of angle a.

$3x - 10 + 2x + 15 = 180$

$5x + 5 = 180$

$-5 \quad -5$

$5x = 175$

$\frac{5x}{5} = \frac{175}{5}$

$x = 35$

$3x - 10$

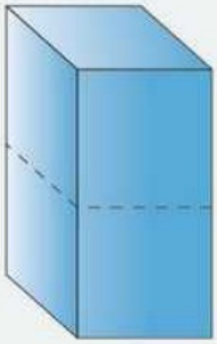
$3(35) - 10$

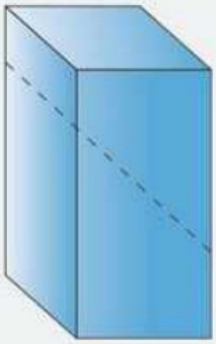
$95°$

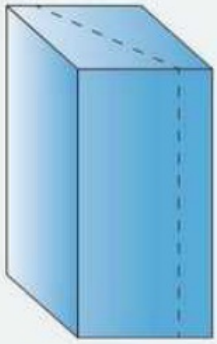
Day 7 – Slicing Three-Dimensional Shapes

Some interesting shapes result when you cut prisms from different angles. Suppose that you start with a square prism like those shown below and make one cut in the directions indicated by the dashed lines. Answer the following questions for each prism.

- What are the shapes of the two resulting figures?
- What are the shapes of the faces of the figures?
- How many vertices, edges, and faces does each figure have?

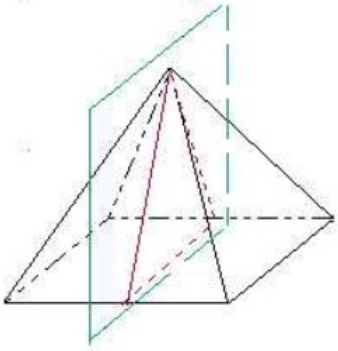
1. 

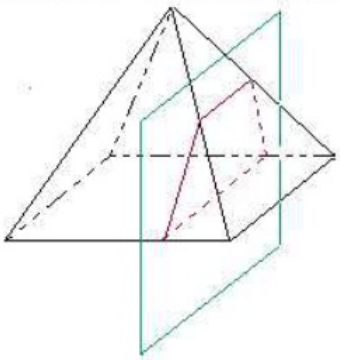
2. 

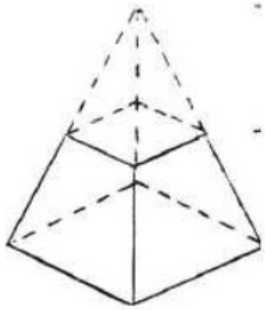
3. 

4) Slice the sphere below. What are the resulting faces?

5) Slice the pyramid below as shown. What are the resulting faces?



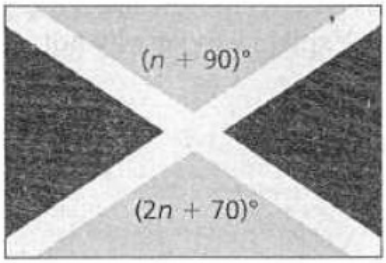




More Algebra in Geometry

1)

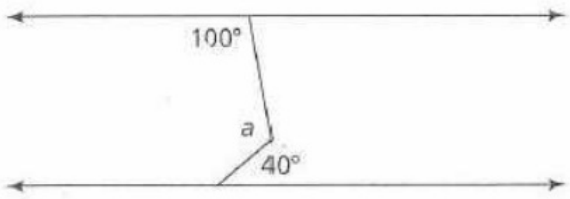
The figure shows the design of the flag of Jamaica.



Find n and the measures of the two angles.

2)

In the figure, the two horizontal lines are parallel.

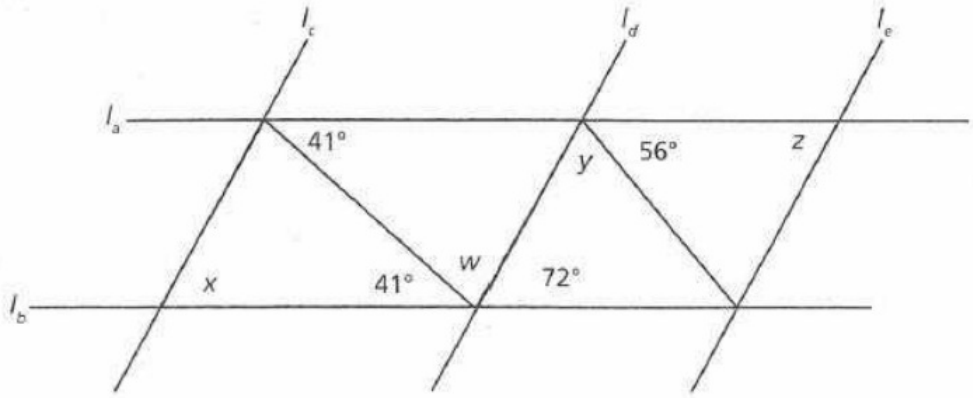


Find the measure of $\angle a$.

3)

Angle 1 is supplementary to angle 2. Angle 2 is vertical to angle 3. Angle 3 is an alternate exterior angle to angle 4. Angle 4 is supplementary to angle 5. How is angle 1 related to angle 5?

For Exercises 26–29, use the diagram below. Lines l_a and l_b are parallel. Lines l_c, l_d, and l_e are parallel.



26. What is the value of w?

27. What is the value of x?

28. What is the value of y?

29. What is the value of z?