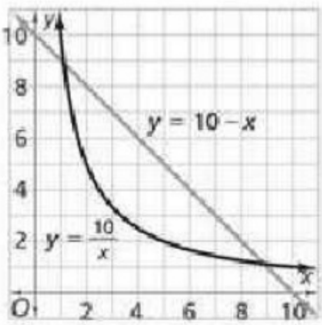
Thinking with Mathematical Models
(TMM)

Day	Topic	Homework	IXL	Grade
1	Inv 1.1	Worksheet 1	Z.3	
2	Inv 1.2	Worksheet 2 and Read Inv 1.3 for tomorrow		
3	Inv 1.3	Worksheet 3	Z.6	
4	Inv 2.1 and Inv 2.2 highlights	Worksheet 4		
5	Inv 2.3	Worksheet 5	Z.7	
6	Inv 2.4	Study for Quiz		
7	Quiz	Worksheet 6	Z.8	
8	Inv 2.5	Worksheet 7		
9	Inv 3.1	Worksheet 8 and Read Inv 3.2 before tomorrow	Z.10	
10	Inv 3.3	Worksheet 9		
11	Functions clarified	Study for Quiz	Z.2	
12	Quiz	Worksheet 10		
13	Inv 4.1	Worksheet 11	Z.14	
14	Inv 4.2	Worksheet 12		
15	Inv 5.1	Worksheet 13	Z.1	
16	Inv 5.2	Worksheet 14		
17	Inv 5.3	Worksheet 15		
18	Practice	Review Packet		
19	Review	Study for Test		
20	Unit Test	none		

Important Concepts	Examples
Mathematical Model An equation or a graph that describes the relationship between two variables. A mathematical model is made by graphing data and finding an equation or a curve to approximate it. A model lets you estimate values between and beyond the data points.	Students model bridge thickness and strength data by: 1. simulating the strength of bridges that have various layers of thickness and collecting data, 2. plotting the data and drawing a line of best fit, 3. finding an equation to model the data (e.g., $y = 8x$), 4. and using the equation to predict the breaking weights for other bridges. For example, using $y = 8x$, a bridge of thickness 3.5 layers can hold a load of 28 pennies.
Linear Relationships and Functions Students have learned how to recognize, represent, and analyze linear relationships. They have learned how to solve linear equations. Students will deepen those understandings in this Unit.	In the equation $y = mx + b$, m indicates the constant ratio $\frac{\text{change in } y}{\text{change in } x}$, which is the slope of the graph. The variable b indicates the y intercept $(0, b)$ of the graph. Students solve linear equations by • approximating (x, y) values in tables and graphs, • undoing the operations involved in linear function calculations by using the properties of equality, and • looking at the associated fact family equations.
Direct Variation Models that can be written in the form $y = kx$.	Students are familiar with direct variation as a special case of a linear function (that is, those with a y-intercept of zero).
Inverse Variation Models that can be written in the form $y = \frac{k}{x}$. The key learning goals for students are first, that an indirect variation gives a non-linear pattern of change and second, that its equation can be written in the form $y = \frac{k}{x}$.	The contrasting graphs of $y = 10 - x$ (line) and $y = \frac{10}{x}$ (curve) demonstrate that dividing by an increasing variable has a different effect than subtracting an increasing variable does. Students are familiar with the formula $A = \ell w$ for finding the area of a rectangle with given length and width. Now, students are asked to look for combinations of length and width that give a fixed area. This leads to the formula $\ell = \frac{A}{w}$. 
Patterns of Association in Numerical Data	Scatter plots can be used to model association between two quantities. Students describe patterns such as clustering, outliers, positive/negative association, and linear/nonlinear association. For linear data, students write a linear model and assess the fit of the model by judging the closeness of the data points to the line.
Patterns of Association in Categorical Data	Students construct and interpret two-way tables of categorical data and use relative frequencies calculated for rows or columns to describe the association between the two variables. In the two-way table below, students look for an association between gender and political party affiliation.

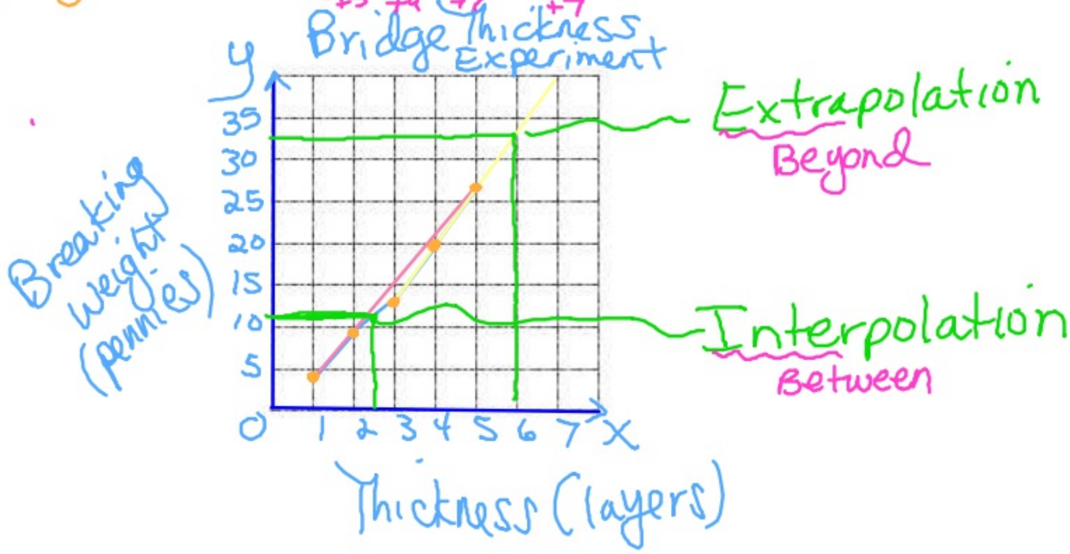
	Democrat	Independent	Republican
Boys	8	4	12
Girls	8	2	6

Date: 9/26/18 Day 1 Inv 1.1

A)

Bridge Thickness (layers)	1	2	3	4	5
Breaking Weight (pennies)	4	9	13	20	27

2.5 11 +5 +4 +7 +7



B) It is close to linear but not exact.
The graph was not a straight line.
It does not have a constant rate of change

C) 2.5 layers 11 pennies

D) 6 layers 34 pennies the last two entries increased by 7 so I added 7 to 27 and got 34.

E) 36 pennies

Date: 9/28/18 Day 2 Inv 1.2

$45 + 14 = 59$ $\frac{59}{2} = 29.5$

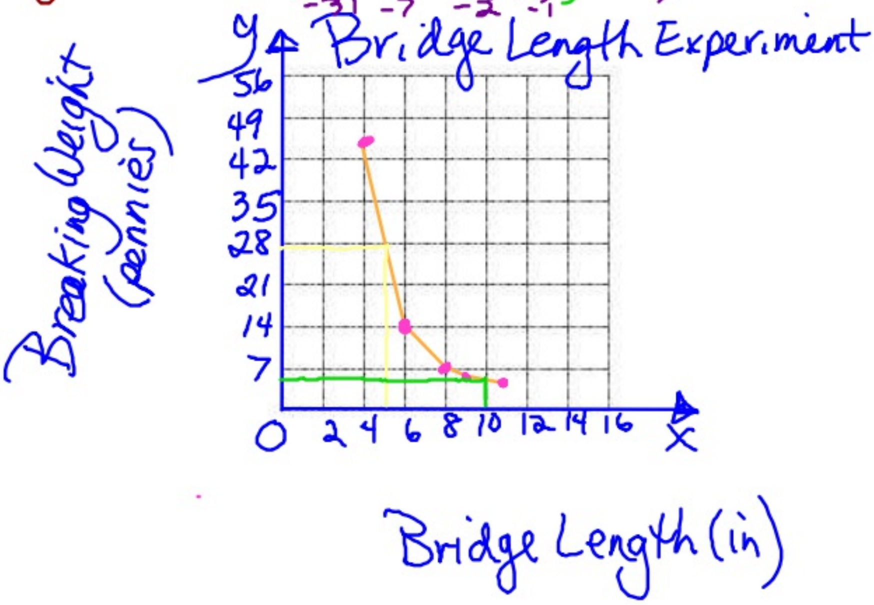
A)

Bridge Length (in)	4	6	8	9	11
Breaking Weight (pennies)	45	14	7	5	4

3 5 10 12

-31 -7 -2 -1 5

$45 - 14 = 31$

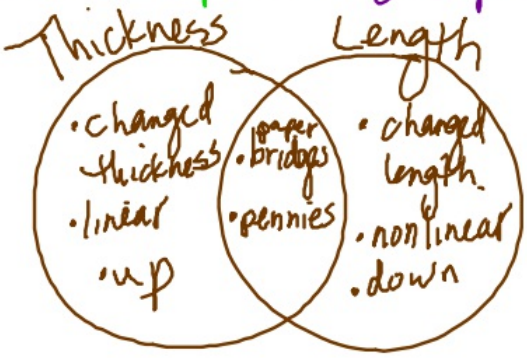


B) This is a nonlinear relationship because the graph is not a straight line and we do not have a constant rate of change in the table.

C) 3 inches 5 inches 10 inches 12 inches

65–90 pennies 27 pennies 5 pennies 2-3 pennies

D)



****BE SURE TO READ THROUGH INV 1.3 BEFORE CLASS TOMORROW!!****

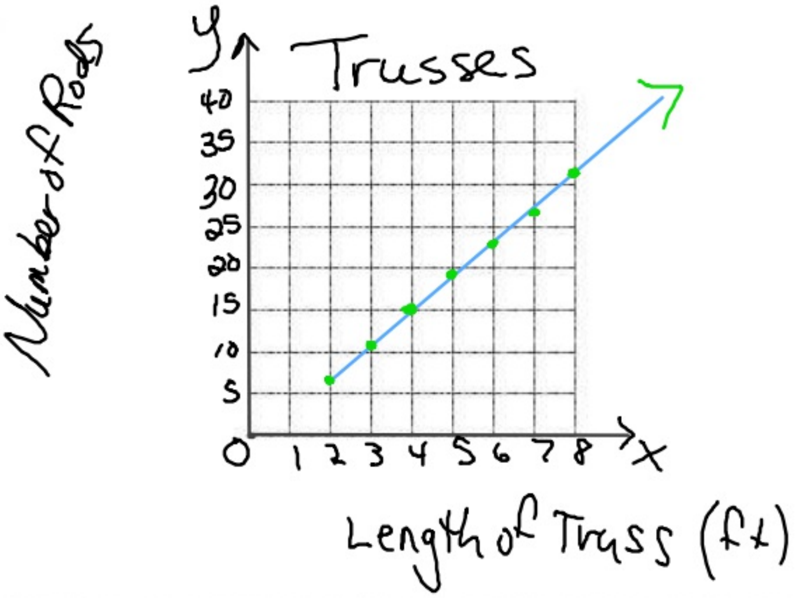
Date: 10/1/18 Day 3 Inv 1.3

A)

Complete the table.

Length of Truss (ft)	0	1	2	3	4	5	6	7	8
Number of Rods	-1	3	7	11	15	19	23	27	31

1) Make a graph.



2) Describe the pattern of change in the number of rods used as the truss length increases.

This is linear. As the length of the truss increases by 1 ft, the number of rods increases by 4.

3) How is the pattern you described shown in the table? In the graph?

table: top row skip count by 1's
bottom row skip count by 4's

graph: line

4) How many steel rods are in a truss $\boxed{\text{50 feet}}$ long overall?

Let x = length of truss (ft)
y = number of rods

$y = mx + b$

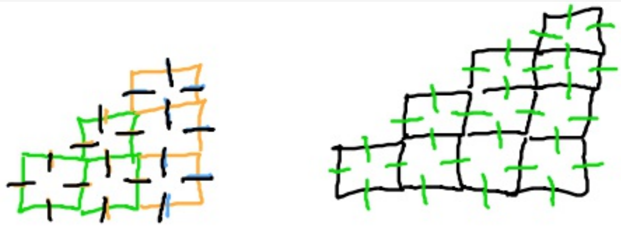
$y = 4x - 1$

$x = 50$ $y = 4(50) - 1$

$y = 200 - 1$

$\boxed{y = 199 \text{ rods}}$

B)

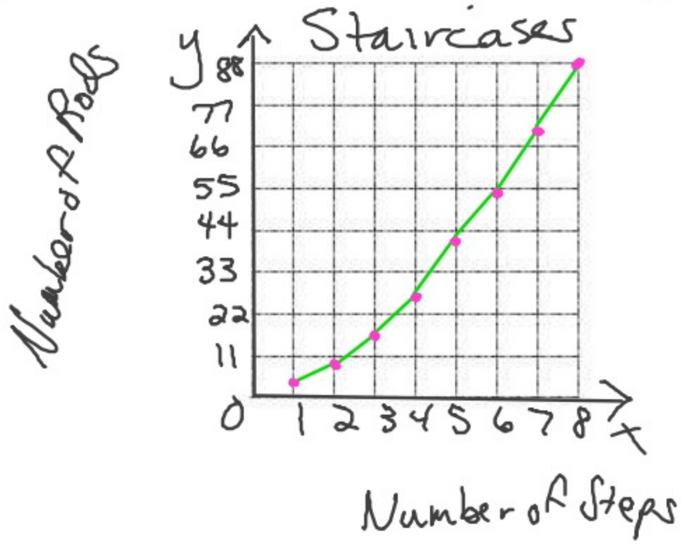


1) Complete the table.

Number of Steps	1	2	3	4	5	6	7	8
Number of Rods	4	10	18	28	40	54	70	88

+6 +8 +10 +12 +14 +16 +18

2) Make a graph.



3) Describe the pattern of change in the number of rods used as the number of steps increases.

This is nonlinear
As the number of steps increases by 1, the number of rods are going up slowly at first then more quickly

4) How is the pattern you described shown in the table? In the graph?

table: there is no constant rate of change. (The change increases by 2 each time)

graph: it is not a line
distances between points is not the same

5) How many steel rods are in a staircase frame with 12 steps?

8	9	10	11	12
88	108	130	154	180

+18 +20 +22 +24 +26

154
+26

$\boxed{\text{180 rods}}$

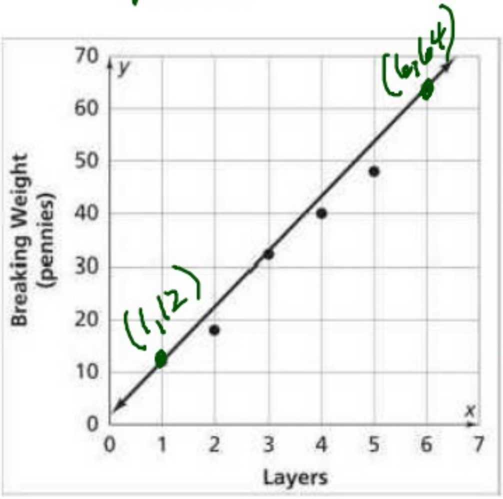
Date: 10/2/18 Day 4 Inv 2.1

Function: Every input (x value) has exactly one output (y value) (function rule = equation)

Residual: the difference between actual data (data collected) points and predicted data points (use the line)

A) 1) vertical gap between point and line.

B)



not a good line of best fit

there are 3 points below and none above.

actual data

Number of Layers	1	2	3	4	5	6
Breaking Weight (pennies)						
Actual	12	18	32	40	48	64
Predicted by Model $y = 10.4x + 1.6$	12	22.4	32.8	43.2	53.6	64
Residual (actual – predicted)	0	-4.4	-.8	-3.2	-5.6	0

→ Sub into equation

10.4(1)+1.6 10.4(2)+1.6 10.4(3)+1.6 10.4(4)+1.6

Describe the residuals.

They are all negative or 0. Not a good line of best fit.

To have a good line of best fit you want equal numbers of positive and negative residuals

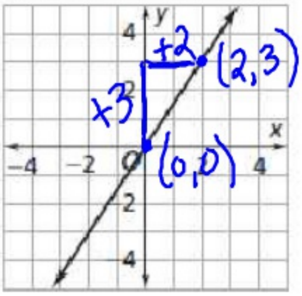
Inv 2.2

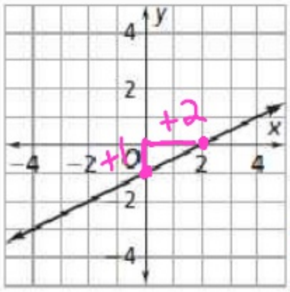
A. For the functions with the graphs below, find the slope and y-intercept. Then write an equation for each line in the form $y = mx + b$.

$b=0$

$m=\frac{3}{2}=1.5$

$\boxed{y=1.5x}$

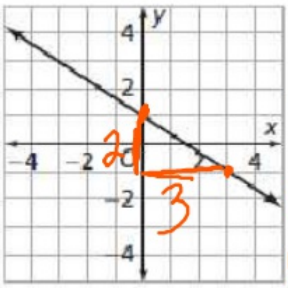
1. 

2. 

$b=-1$

$m=\frac{1}{2}$

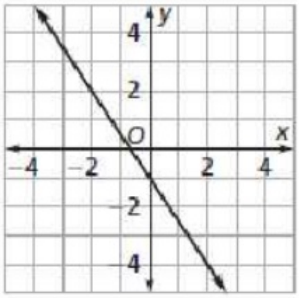
$y=\frac{1}{2}x-1$

3. 

$b=1$

$m=-\frac{2}{3}$

$y=-\frac{2}{3}x+1$

4. 

B. 1. Write a linear equation for the function represented in each table.

$\rightarrow +1$

a.

x	−2	−1	0	1	2
y	−1	1	3	5	7

$\rightarrow +2$

$b=3$

$m=2$ $y=2x+3$

b.

x	−6	−2	2	6	10
y	−4	−2	0	2	4

Profit = Income - Expenses

Date: 10/3/18 Day 5 Inv 2.3

A)
1) $I = \underline{25}n + \underline{0}$ $\boxed{I = 25n}$

2) $P = 25n - 500$

3) $B = \underline{^-350}m + \underline{4500}$

$m = \frac{\Delta y}{\Delta x} = \frac{75}{5} = 15$

B)

The table below shows a plan for group prices of a *Tree Top Fun* franchise.

0	n **Number in Group**	1	2	3	4	5	10	~~10~~ 15	20
60	A **Admission (dollars)**	75	90	105	120	135	210	285	360

+5

+15 +15 +15 +15 +75 +75 +75

1. Explain how you know the relationship between the admission fee for a group and the number of people in the group is linear.

It is linear because it has a constant rate of change.

2. What are the slope and *y*-intercept of the graph of the data?

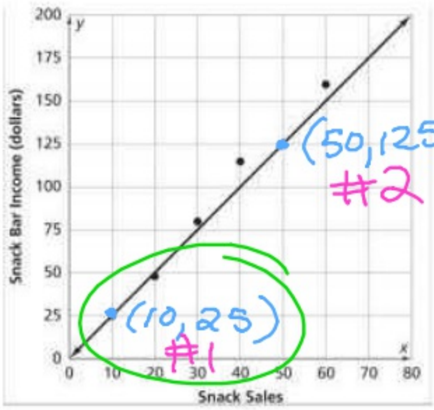
$m = 15$ $b = 60$

3. What equation relates rental charge *A* to number *n* in the group?

$y = 15x + 60$ $\boxed{A = 15n + 60}$

C)

The graph below shows the income from snack sales at *Tree Top Fun* for six different days.



$y = mx + b$

$25 = 2.5(10) + b$

$25 = 25 + b$

$0 = b$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{125 - 25}{50 - 10}$

$= \frac{100}{40} = 2.5$

Write an equation for the linear model in the graph.

x	10	50
y	25	125

$y = 2.5x + 0$

$\boxed{y = 2.5x}$

Date: 10/4/18 Day 6 Inv 2.4

$c = 0.15t + 2.50$

Let c = charge ($)
Let t = time (min)

$y = mx + b$ $\quad m = \frac{\Delta y}{\Delta x}$ $ / min

A)
1) Explain what the numbers mean for this situation.

0.15 = slope $.15 per minute to rent the canoe

2.50 = y-int. $2.50 no matter what

per
no matter what

2) How much does it cost to rent a canoe for 25 minutes?

t = 25 min

$C = .15t + 2.50$
$C = .15(25) + 2.50$
$C = \boxed{\$6.25}$

3) A customer is charged $9.25. How long did he use the canoe?

C = 9.25

$9.25 = .15t + 2.50$
$-2.50 \quad -2.50$
$6.75 = .15t$
$\frac{6.75}{.15} = \frac{.15t}{.15}$

$\boxed{t = 45 \text{ min}}$

4) A customer has $6 to spend. How long can she canoe?

C = 6

$6 = .15t + 2.50$
$-2.50 \quad -2.50$
$3.50 = .15t$
$\frac{3.50}{.15} = \frac{.15t}{.15}$

$\boxed{t = 23.\overline{3} \text{ min}}$

E) $c = 4 + 0.10t$ Let c = charge ($) Let t = time (min)

1) What is the charge to rent a paddle boat for 20 minutes?

t = 20 min

$C = 4 + .10(20)$
$C = \boxed{\$6}$

2) A customer at River Fun is charged $9. How long did the customer use a paddle boat?

C = $9

$9 = 4 + .10t$
$-4 \quad -4$
$5 = .10t$
$\frac{5}{.10} = \frac{.10t}{.10}$

$\boxed{t = 50 \text{ min}}$

3) Suppose you want to spend $12. How long could you use a paddle boat?

$12 = 4 + .10t$
$-4 \quad -4$
$8 = .10t$
$\frac{8}{.10} = \frac{.10t}{.10}$

$\boxed{t = 80 \text{ min}}$

B) How could Rashida use the graph to answer questions in part A?

C) How could Serena use the table to answer the questions in part A?

D) 2) inequality: $0.15t + 2.50 \leq 6$ Solve the inequality and graph it on a number line. What does this tell you about the situation?

*****Day 7 is a quiz*****

Date: _____ Day 8 Inv 2.5

A)

Saturday Resort Attendance

Probability of Rain (%)	0	20	40	60	80	100
Big Fun Attendance	1,000	850	700	550	400	250
Get Reel Attendance	300	340	380	420	460	500

Equation Model for Big Fun:

Equation Model for Get Reel:

B)

Date: 10/10/18 Day 9 $A = lw$ Inv 3.1

$\boxed{A = 24\,in^2}\ w = ?$
$l = 2\,in$

Complete the table and plot your data.

$w = \frac{24}{l}$

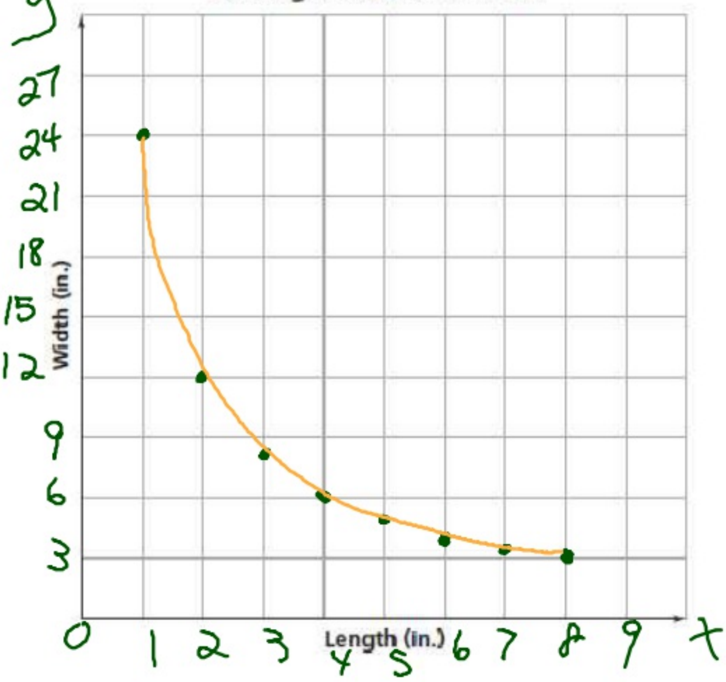
$1 \cdot (?) = 24$
$2 \cdot (?) = 24$
$3 \cdot ? = 24$
$\frac{5 \cdot ?}{5} = \frac{24}{5}$
$? = \frac{24}{5}$

Rectangles With Area 24 in.²

Length (in.)	1	2	3	4	5	6	7	8
Width (in.)	24	12	8	6	4.8	4	3.4	3

$\frac{24}{7} = 3\frac{3}{7}$

Rectangles With Area ~~32~~ 24 in.²



3) Describe the pattern of change in the width of the rectangle as the length increases. Is this relationship linear?

This is not a linear relationship.
As the length increases by 1 in, the width decreases quickly at first, then more slowly.

4) Write an equation.

Let x = length (in)
Let y = width (in)

$24 = xy$

$y = \frac{24}{x}$

Inverse Relationship
X is in the denominator

Read Inv 3.2 on p. 63-65

Date: 10/11/18 Day 10 Inv 3.3

Inverse variation: (Inverse Relationship) the product of x and y is a constant value ($k=xy$) the independent variable (x) is in the denominator $y=\frac{k}{x}$

A) 1) Complete the table.

n Number of students	25	50	75	100	125	150	175	200	~~250~~
c Cost per student ($)	30	15	10	7.50	6	5	4.29	3.75	~~3~~

$\frac{750}{25}$ $\frac{750}{50}$

$\frac{750}{250}=3$

$\frac{750}{175}=4.285714286$

2) Describe the pattern.

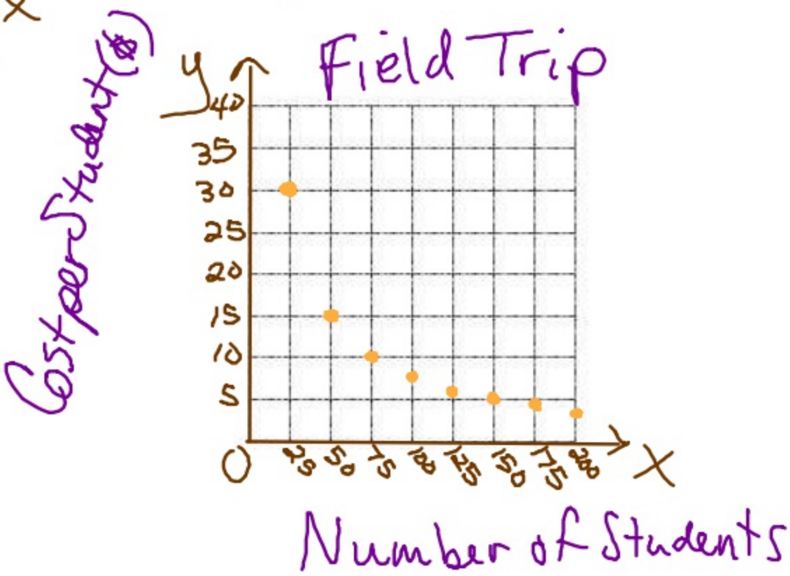
As the number of students increases by 25, the cost per student decreases quickly at first then more slowly.

3) Write an equation (check the book for the variables!).

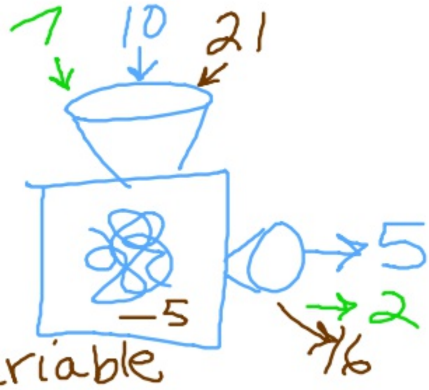
$750=xy$ $750=nc$

$y=\frac{750}{x}$ $c=\frac{750}{n}$

4) Make a graph.



Date: 10/16/18 Day 11 Functions Clarified

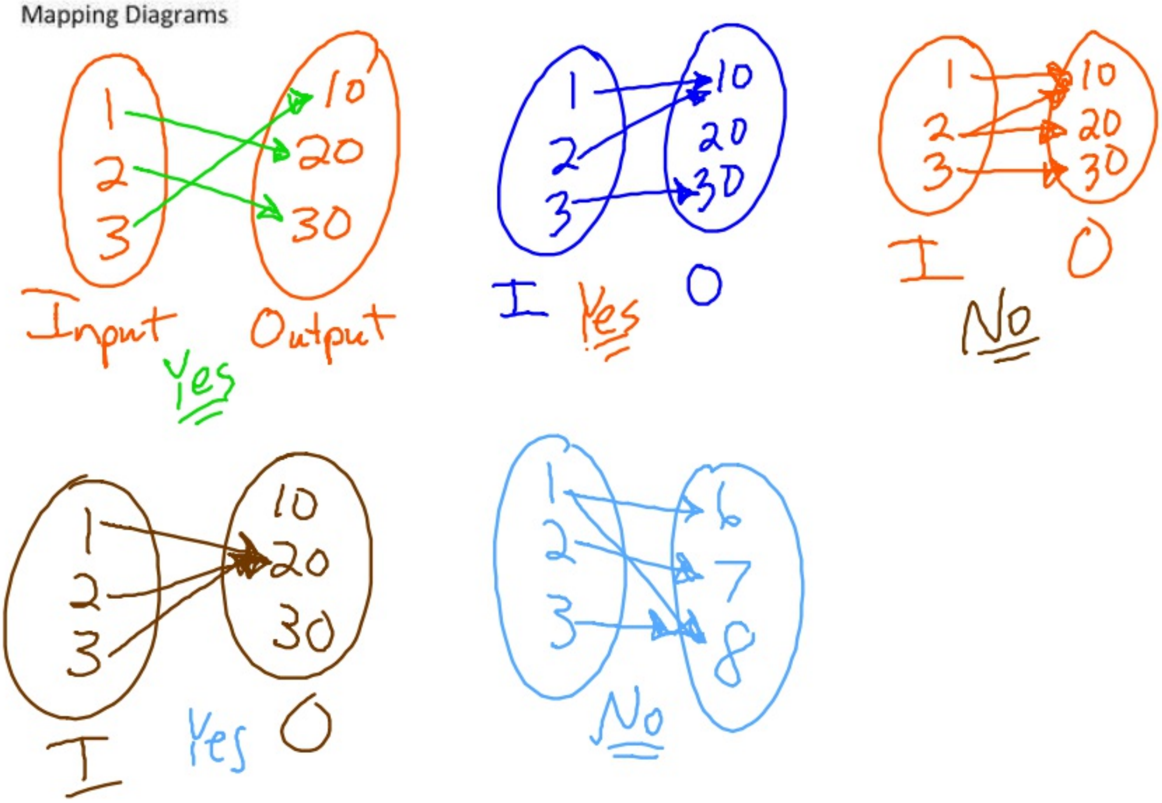


Function: a relation where every input has exactly one output.

Input: x values / independent variable (domain: list of all inputs)

Output: y values / dependent variables (range: list of all outputs)

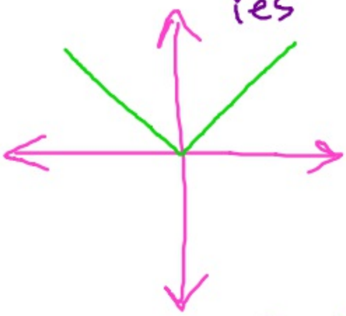
Mapping Diagrams



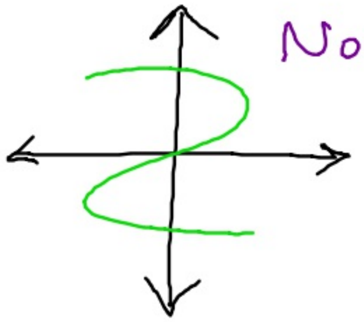
Graphs

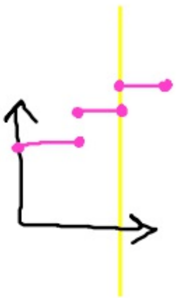
Yes

Yes



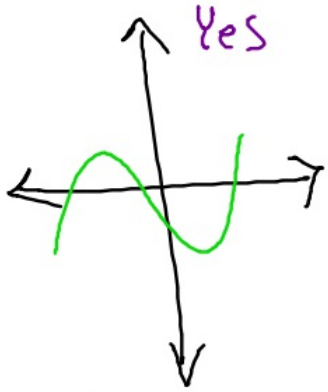
No





In order for a graph to be a function, it must pass the vertical line test. (never more than 1 point on any vertical line)

Yes



Tables

Yes

x	0	1	2	3
y	5	6	7	7

No

x	4	4	4	4
y	5	6	7	8

Yes

x	0	1	2	3
y	5	5	5	5

No

x	0	1	0	1
y	5	6	7	8

x values should not repeat.

$\{(0,5)(1,6)(0,7)(1,8)\}$ No

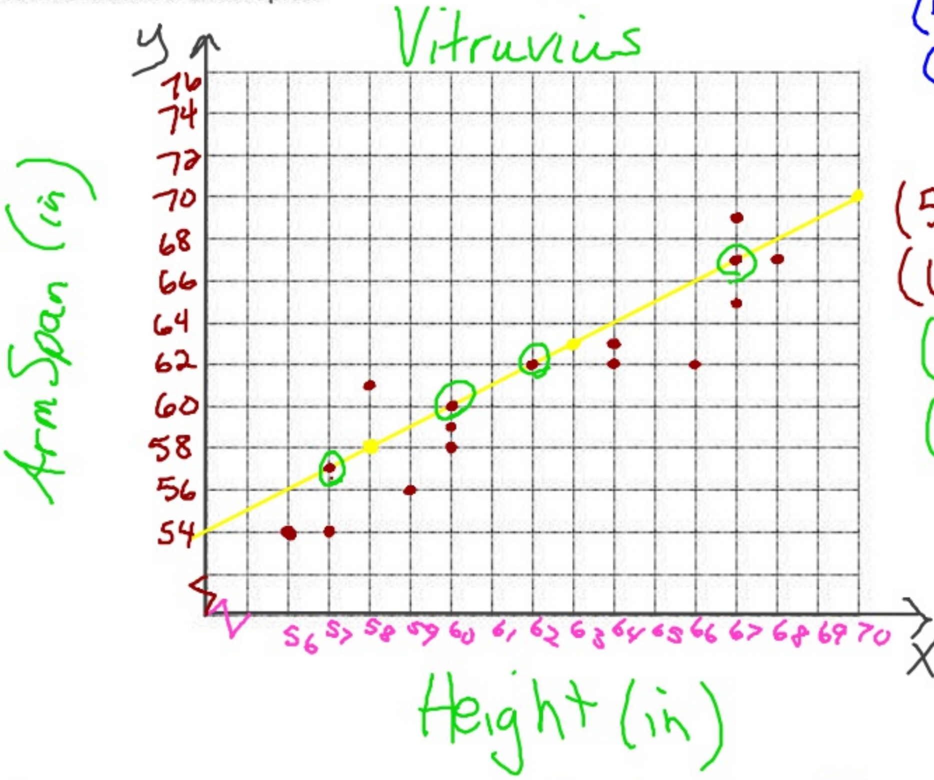
$\{(1,3)(2,4)(3,3)(4,4)\}$ Yes.

****Day 12 is a quiz****

Date: 10/18/18 Day 13 Inv 4.1

Scatter plot: a graph where the points are simply plotted but not connected (they are "scattered" on the grid)

A) 1) Plot the data in a scatter plot.



(58,58) (70,70) > Vitruvius

(57,57) (62,62) (60,60) (67,67) } y=x

2) The graph shows that height and arm span are almost equal.

3) equation: y=x

a) Graph the equation on your scatter plot. ✓

b) Which data points does your line pass through? see above

B) 1) Where would you plot the point? Would the point be on, above or below your line?

Plot the point above y=x because arm span > height

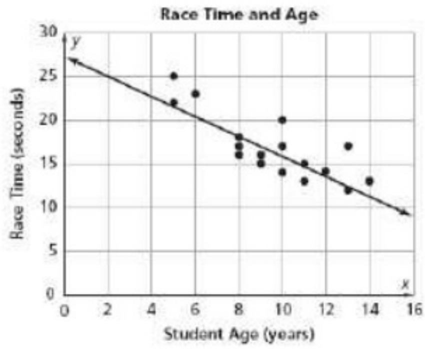
2) Does the data support the claim that arm span and height are roughly equal?

D) 1) Do you think the T Rex data points fit the pattern that arm span and height are roughly equal?

2) If you plot the data point, would it be on, above, or below the line?

Date: _____ Day 14 Inv 4.2

Student Age (years)	5	5	6	8	8	8	9	9	10	10	10	11	11	12	13	13	14
Race Time (seconds)	25	22	23	18	16	17	15	16	17	20	14	15	13	14	17	12	13



A) 1) What is the approximate slope?

2) How does the slope help you understand the relationship between age and race time?

3)

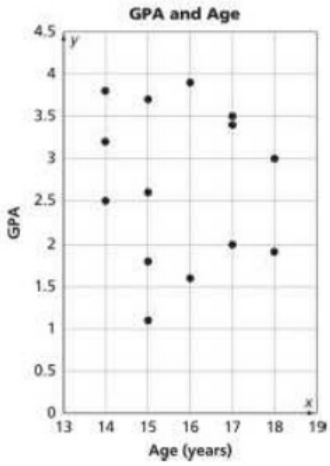
4)

B) 1) State the coordinates of two data points that do not seem to fit the model.

2)

C)

Student Age (years)	14	14	14	15	15	15	15	16	16	17	17	17	18	18
GPA (0.4–4.0)	2.5	3.2	3.8	1.8	2.6	3.7	1.2	1.6	3.9	2.0	3.4	3.5	1.9	3.0



1)

2)

3)

Date: 10/24/18 Day 15 Inv 5.1

	Prefer Wood		Prefer Steel	
Age ≤ 40 years	45	+	60	= 105
Age > 40 years	+ 15	+	+ 20	= 35
	60		80	

A) Is each statement true or false? Be sure to explain your answer!!!

1) $\frac{105}{35} = 3$ $15(3) = 45$ $20(3) = 60$

$\frac{3}{7} = \frac{15}{35} = \frac{45}{105} = \frac{3}{7}$ False. They didn't ask the same size groups. Both groups had $\frac{3}{7}$ of the group pick wood.

2) $\frac{4}{7} = \frac{60}{105} = \frac{20}{35} = \frac{4}{7}$ False

3) $\frac{3}{4}(80) = 60$ True

4) False

5) False

B) Be sure to include the data from part A

1) How many riders would you expect on the wood frame coaster and how many on the steel frame coaster?

2) Complete the table.

	Prefer Wood	Prefer Steel	Total
Age ≤ 40 years			420
Age > 40 years			210
Total			

C) Which would you recommend? Explain your choice.

Date: _____ Day 16 Inv 5.2

Looking at the table on page 115, decide if each statement is true or false. **Be sure to justify your answer!!**

A)

1) Girls and boys are equally likely to become democrats.

2) Boys are more likely than girls to Independents.

3) Boys are more likely than girls to be Republicans.

4) Girls are only half as likely as boys to be Republicans.

B) 1) Complete the table.

	Democrat	Independent	Republican	Totals
Boys	8	4	12	
Girls	8	2	6	
Totals				

2) Do any of these totals change your answers to part A? Explain.

C) 1) Complete the table.

	Democrat	Independent	Republican
Boys			
Girls			

2) Do the percent calculations change your answers to part A?

Date: ______ Day 17 Inv 5.3

A) Make a table. This time, be sure to add a row and column for percents as well as totals.

B) Decide whether each statement is true or false. Be sure to explain.

1)

2)

3)

4)